23. M. A. Anisimov, S. B. Kiselev, and I. G. Kostukova, Proc. ASME Ann. Meeting (HT-53), Anaheim, California (1986), pp. 1-8.
24. M. A. Anisimov, S. B. Kiselev, I. G. Kostyukova, and V. B. Nagaev, The Thermophysical Parameters of Substances and Materials, The State Standard Specimen Register: Physical Constants and Properties of Materials [in Russian], No. 27, Moscow (1988), pp. 5-13.
25. R. T. Ellington, B. E. Eakin, J. D. Parent, et al., Thermodyn. and Transp. Properties of Gases, Liquids and Solids, McGraw-Hill, New York–Toronto–London (1959), pp. 180-194.
26. I. Wichterle, S. Chappelear, and R. Kobayashi, J. Comput. Phys., 7, No. 3, 606-620 (1971).
27. J. C. Le Guillou and J. Zinn-Justin, J. Physique Lett., 46, 137-141 (1985).
28. V. A. Rabinovich and Yu. E. Sheludyak, Inzh.-Fiz. Zh., 51, No. 5, 758-759 (1986).

PHASE TRANSITIONS IN SLOT SYSTEMS FOR SMALL Kn NUMBERS

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The combined problems of sublimation of a thin deposit on the walls of a narrow slot and of freezing-on of a desublimate layer on the walls are considered.

Weakly nonequilibrium phase transitions on the surfaces of parallel plates forming slot devices with the process rate depending substantially on the phase resistance were investigated in [1] by using the methods of molecular kinetics. Our aim is to analyze the processes whose rate is determined almost entirely by the hydraulic resistance of the slot system and the thermal resistance of the walls, as the effect of phase resistance is negligible.

In considering viscous and molecular-viscous sublimate flows, which correspond roughly to the $Kn \ll 0.01$ and $0.01 < Kn < 0.1$ ranges, we can use with an accuracy sufficient for technological purposes the ordinary Navier–Stokes equations. However, corrections for slippage, thermal slip (creep), and the temperature jump must be introduced in the boundary conditions. If the height of the slot device is small in comparison with its plane dimensions ($\gamma \ll 1$), the continuity and momentum equations can be conveniently represented in dimensionless form:

$$\frac{\partial}{\partial\zeta}\left(\frac{\Pi}{\Theta}w\right) = -\nabla\left(\frac{\Pi}{\Theta}\bar{\mathbf{u}}\right), \tag{1}$$

$$\begin{gathered}\frac{\partial\Pi}{\partial\zeta} = \gamma^2\bar{\mu}\frac{\partial^2 w}{\partial\zeta^2} - \gamma^2 \mathrm{Re}_*\frac{\Pi}{\Pi_*\Theta}\left[(\bar{\mathbf{u}}\nabla)w + w\frac{\partial w}{\partial\zeta}\right] + \\ + \gamma^4\bar{\mu}\nabla^2 w - \left(\frac{\bar{\mu}}{3} + \bar{\mu}'\right)\gamma^3\frac{\partial}{\partial\zeta}\left[\frac{\bar{\mathbf{u}}\nabla(\Pi/\Theta) + w\partial(\Pi/\Theta)/\partial\zeta}{\Pi/\Theta}\right],\end{gathered} \tag{2}$$

$$\frac{\partial^2\bar{\mathbf{u}}}{\partial\zeta^2} = \frac{1}{\bar{\mu}}\nabla\Pi + \frac{\mathrm{Re}_*\Pi}{\Pi_*\Theta\bar{\mu}}\left[(\bar{\mathbf{u}}\nabla)\bar{\mathbf{u}} + w\frac{\partial\bar{\mathbf{u}}}{\partial\zeta}\right] - \gamma^2\nabla^2\bar{\mathbf{u}} + \left(\frac{1}{3} + \frac{\bar{\mu}'}{\bar{\mu}}\right)\gamma^2\nabla\left[\frac{\bar{\mathbf{u}}\nabla(\Pi/\Theta) + w\partial(\Pi/\Theta)/\partial\zeta}{\Pi/\Theta}\right]. \tag{3}$$

The energy equation will be needed only to demonstrate that, in these processes, the variation of the sublimate temperature occurs almost exclusively along the walls of the slot device. However, in order to avoid the impression that some of the neglected caloric effects

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might have physical significance, we shall write this equation in a rather rigorous form. In terms of dimensional variables, it is given by [2]

$$\operatorname{div}(\lambda \operatorname{grad} T) = \rho c_p \bar{\mathbf{v}} \operatorname{grad} T - \frac{\mu}{2}\left\{\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)^2 + \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)^2\right.$$

$$\left. + \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)^2 + 4\left[\left(\frac{\partial v_x}{\partial x} - \frac{1}{3}\operatorname{div}\bar{\mathbf{v}}\right)^2 + \left(\frac{\partial v_y}{\partial y} - \frac{1}{3}\operatorname{div}\bar{\mathbf{v}}\right)^2 + \left(\frac{\partial v_z}{\partial z} - \frac{1}{3}\operatorname{div}\bar{\mathbf{v}}\right)^2\right]\right\} - \mu'(\operatorname{div}\bar{\mathbf{v}})^2,$$

while, after passage to dimensionless variables, we write it in a form suitable for analysis:

$$\frac{\partial}{\partial \zeta}\left(\bar{\lambda}\frac{\partial \Theta}{\partial \zeta}\right) = \mathrm{Pe}_* \frac{\Pi}{\Theta}\left(\bar{\mathbf{u}}\nabla\Theta + w\frac{\partial \Theta}{\partial \zeta}\right) - \frac{\gamma_c - 1}{2}\mathrm{Pr} M_*^2 \bar{\mu}\left[\left(\frac{\partial u_\xi}{\partial \zeta}\right)^2 + \left(\frac{\partial u_\eta}{\partial \zeta}\right)^2\right] + O(\gamma^2). \tag{4}$$

The boundary conditions which must be satisfied at the inside surfaces of the slot walls, written in terms of dimensional variables, are given by

$$\bar{v}_\tau = \mp \frac{2-\sigma_0}{\sigma_0} l \frac{\partial \bar{v}_\tau}{\partial z} + \frac{3}{4}\frac{\mu}{\rho T}(\operatorname{grad} T)_\tau,\quad \rho\bar{v}_z = \mp J^{(\pm)},$$

$$T = T_w \mp \frac{2-\alpha}{\alpha}\frac{\gamma_c}{\gamma_c + 1}\frac{l}{\mathrm{Pr}}\frac{\partial T}{\partial z} \quad \text{for} \quad z = \pm h.$$

After passage to dimensionless variables, we obtain

$$\bar{\mathbf{u}} = \mp \frac{2-\sigma_0}{\sigma_0}\frac{1{,}26\gamma\sqrt{\Theta}\,\bar{\mu}}{\sqrt{\gamma_c}\,M_*\Pi}\frac{\partial \bar{\mathbf{u}}}{\partial \zeta} + \frac{3}{4}\frac{\gamma^2\bar{\mu}}{\gamma_c M_*^2 \Pi}\nabla\Theta,\quad w = \mp\frac{\Theta}{\Pi}J_0^{(\pm)}, \tag{5}$$

$$\Theta = \Theta_w^{(\pm)} \mp \frac{2-\alpha}{\alpha}\frac{2\gamma_c}{\gamma_c + 1}\frac{1{,}26\gamma\sqrt{\Theta}\,\bar{\mu}}{M_*\mathrm{Pr}\sqrt{\gamma_c}\,\Pi}\frac{\partial \Theta}{\partial \zeta} \quad (\zeta = \pm 1). \tag{6}$$

It is assumed that sublimation (condensation) of the same substance occurs on both inside surfaces of the slot. If $\gamma^2 \ll \mathrm{Re}_*$ or $\gamma^2 = O(\mathrm{Re}_*)$, then, considering that $\mathrm{Pr} \leq 1$ for gases, we use (4) to provide the following estimate for the ratio of the temperature drop δT_h along the slot height to the temperature drop δT_L in the symmetry plane: $\delta T_h/\delta T_L = O(\mathrm{Re}_*) + O(M_*^2)$. In view of (6), the role of the temperature jump $\Theta - \Theta_w$ can be estimated by using the expression

$$(\Theta - \Theta_w)/\delta\Theta_L = O(\mathrm{Kn}\,\mathrm{Re}_*) + O(\mathrm{Kn}\,M_*^2).$$

Consequently, if $\mathrm{Re}_* \ll 1$ and $M_*^2 \ll 1$, any transverse temperature drop is negligible, and we assume that phase transitions occur under quasi-equilibrium conditions in the slot device, i.e., the Clausius–Clapeyron expression relating the pressure, which, according to (2), is independent of the transverse coordinate ζ with an accuracy to $O(\gamma^2)$, to the sublimate temperature, holds:

$$\Theta = \Theta_s(\Pi_1) = (1 - \omega \ln \Pi_1)^{-1}, \tag{7}$$

which means that saturated vapor flows everywhere in the slot. It should be emphasized that, in slot gas dynamics, for $\gamma^2 \ll 1$, very large relative pressure drops $\delta p = O(p_*)$ can be associated with small values of the parameter $M_*^2 \ll 1$, since

$$M_*^2 = \frac{V^2}{a_*^2} = \frac{\gamma^2 p_*^2 h^2}{(\gamma_c - 1)c_p T_* \mu_*^2}.$$

Thus, if we neglect $O(\gamma)$ and $O(\mathrm{Re}_*)$, it follows from (2), (3), (5), and (6) that

$$\bar{u} = -\frac{1}{2}\left[\frac{1-\zeta^2}{\bar{\mu}(\Theta)} + \frac{2-\sigma_0}{\sigma_0}\frac{2{,}52\gamma\sqrt{\Theta}}{\sqrt{\gamma_c}M_*\Pi}\right]\nabla\Pi + \frac{3}{4}\frac{\gamma^2\bar{\mu}(\Theta)}{\gamma_c M_*^2\Pi}\nabla\Theta, \tag{8}$$

$$w = \mp\frac{RT_*L}{p_*hV}\frac{\Theta_s(\Pi_1)}{\Pi_1}J^{(\pm)} \quad \text{for} \quad \zeta = \pm 1. \tag{9}$$

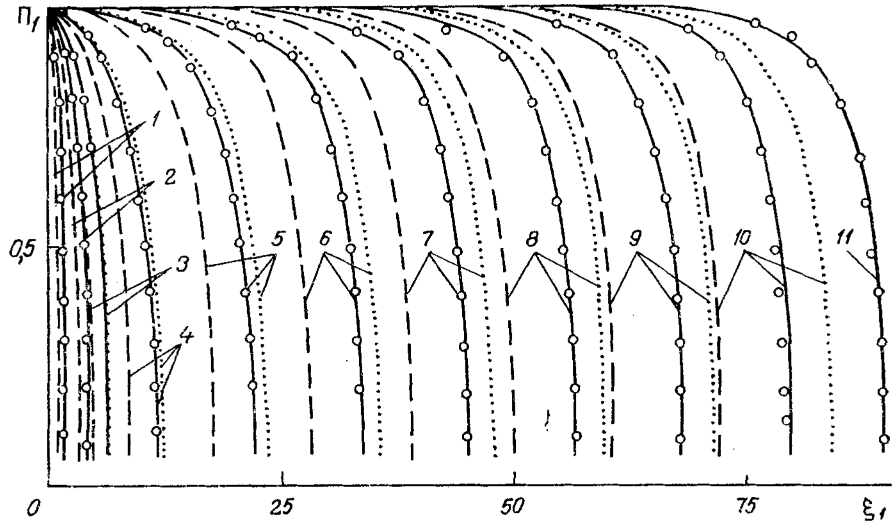

Fig. 1. Pressure distribution in a slot system during sublimation of water ice from the inside walls (ω = 0.04457): solid curves: $\nu = 1$ and $K_{02} = K_{03} = 0$; dotted curves: $\nu = 1$, $K_{02} = 0.1$, and $K_{03} = 0$; dashed curves: $\nu = 0$ and $K_{02} = K_{03} = 0$; 1) $\delta\Theta_w = 1$; 2) 10^{-1}; 3) $5\cdot10^{-2}$; 4) 10^{-2}; 5) 10^{-3}; 6) 10^{-4}; 7) 10^{-5}; 8) 10^{-6}; 9) 10^{-7}; 10) 10^{-8}; 11) 10^{-9}.

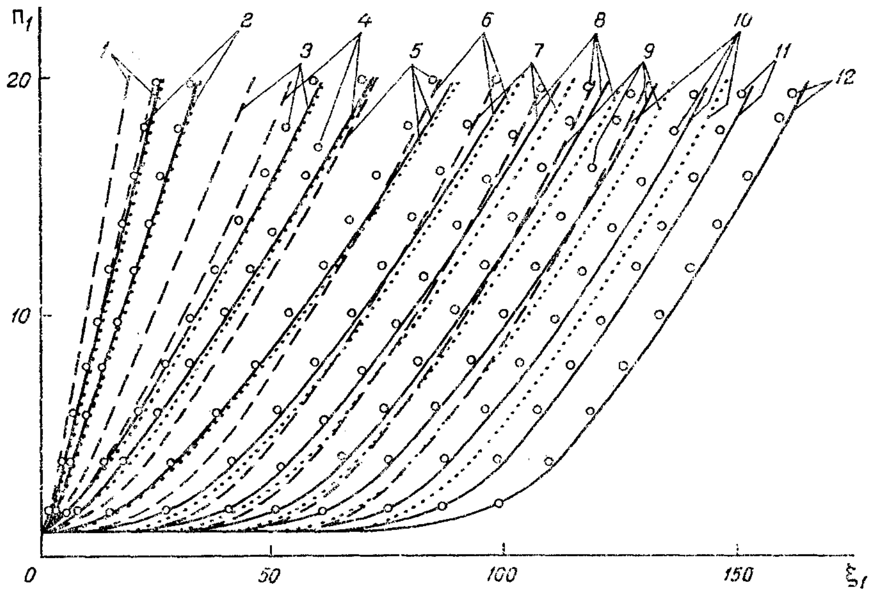

Fig. 2. Pressure distribution in a slot system during water vapor desublimation (ω = 0.04457) at its inside walls; solid curves: $\nu = 1$ and $K_{02} = K_{03} = 0$; dotted curves: $\nu = 1$ $K_{02} = 0.1$, and $K_{03} = 0$; dashed curves: $\nu = 0$ and $K_{02} = K_{03} = 0$; 1) $\delta\Theta_w = -1$; 2) -0.5; 3) -10^{-1}; 4) $-5\cdot10^{-2}$; 5) -10^{-2}; 6) -10^{-3}; 7) -10^{-4}; 8) -10^{-5}; 9) -10^{-6}; 10) -10^{-7}; 11) -10^{-8}; 12) -10^{-9}.

After integrating (1) with respect to ζ within the interval (−1, 1) and using expressions (8) and (9), we obtain the relationship

$$\nabla^2\Psi = -\frac{3}{2}\left(J_\Lambda^{(-)} + J_\Lambda^{(+)}\right), \tag{10}$$

$$\Psi(\Pi_1) = \int_1^{\Pi_1} \Phi(\varkappa)\,d\varkappa = \Psi_1 + K_2\Psi_2 + K_3\Psi_3; \quad \Psi_j(\Pi_1) = \int_1^{\Pi_1} \Phi_j(\varkappa)\,d\varkappa;$$

$$\Phi_1(\varkappa) = \frac{\varkappa}{\bar{\mu}[\Theta_s(\varkappa)]\Theta_s(\varkappa)}; \quad \Phi_2(\varkappa) = \frac{1}{\sqrt{\Theta_s(\varkappa)}};$$
$$\Phi_3(\varkappa) = -\bar{\mu}[\Theta_s(\varkappa)]\frac{\Theta_s'(\varkappa)}{\Theta_s(\varkappa)}, \quad \Theta_s' = \frac{d\Theta_s}{d\varkappa}. \tag{10}$$

In view of (7), $\Theta_S(\Pi_1)$ is a logarithmic relationship with a very small coefficient ω, especially if the latent heat of sublimation is large. At the triple point, $\omega = 0.04457$ corresponds to water. Therefore, it can be expected that the effect of nonisothermicity on the viscosity of coefficients and thermal conductivity is not very substantial. In order to estimate this effect, the coefficients $\bar{\mu}$ and $\bar{\lambda}$ can be represented by the approximate power relationship $\bar{\mu} \approx \bar{\lambda} \approx \Theta_1^{\delta}$ with the exponent $\delta \approx 0.8$ for $T \approx 273$ K and $\delta \approx 1$ for $T \ll 273$ K [3]. If we consider that a pressure reduction by a factor of 10.18 lowers the saturated-vapor temperature of water vapor by only 9.34% in comparison with the initial level at the triple point, while a reduction in pressure by a factor of 4.45 lowers this temperature by only 6.23%, it is clear that $\bar{\lambda} \approx \bar{\mu} = 1 + \Delta(\Pi_1)$, where $\Delta(\Pi_1) \ll 1$ for such substances. These considerations might justify substituting unity for the coefficients $\bar{\mu}$ and $\bar{\lambda}$; however, for a higher degree of conclusiveness, we shall continue to consider the temperature dependence $\bar{\mu}(\Theta_S)$.

The solution of the problem is greatly simplified if the right-hand side of Eq. (10), which characterizes the intensity of phase transitions, is a known function of the coordinates, while constant pressure values are assigned at the open sections of the slot device's contour. This is because (10) is Poisson's equation in this case, and determination of the Ψ function consists in solving the (generally mixed linear problem with homogeneous boundary conditions of the second kind along the closed sections of the slot device's contour and boundary conditions of the first kind at the open sections of the contour. Having found the Ψ function, we determine the pressure field in the slot by using the $\Psi_j(\Pi_1)$ diagrams, plotted on the basis of Eqs. (10), and the coefficients K_2 and K_3, calculated for the above conditions.

If the local intensity of phase transitions is a function, even if a linear one, of the sublimate temperature, then, due to the nonlinear relationship (7) between Π_1 and Θ, a nonlinearity would arise in the right-hand side of Eq. (10), which would be intensified considerably by the fact that the relationship $\Psi(\Pi_1)$ is also nonlinear, and the transition from Ψ and Π_1 can no longer be effected after the boundary-value problem has been solved. In this case, along with the nonlinearity of the right-hand side, there is also the nonlinearity of the differential operator, which is due to the compressibility of the sublimate, nonisothermicity of the flow, and the slippage effect.

Assume that heat is supplied to (removed from) the slot device's inside surfaces at which phase transitions occur through relatively thin walls. The outside surfaces of these walls are maintained at the constant temperatures $T^{(+)}$ and $T^{(-)}$, while variations in the thermal resistance caused by sublimation of the deposit can be neglected. Then, the dimensionless intensity $J^{(\pm)} = (T_*/\Lambda)(\lambda^{(\pm)}/\delta^{(\pm)})\delta\Theta_W^{(\pm)}$ and Eq. (10) are rewritten as follows:

$$\nabla_1(\Phi_0\nabla_1\Pi_1) - (\omega\ln\Pi_1)/(1-\omega\ln\Pi_1) + \delta\Theta_w = 0. \tag{11}$$

Here $\Phi_0 = \Phi_{01} + \Phi_{02} + \Phi_{03}$, where $\Phi_{01} = \Pi_1(1 - \omega\ln\Pi_1)^{1+\delta}$; $\Phi_{02} = K_{02}\sqrt{1 - \omega\ln\Pi_1}$; $\Phi_{03} = -K_{03}\cdot(\omega/\Pi_1)(1 - \omega\ln\Pi_1)^{-1-\delta}$.

As an illustration, we shall consider two very simple boundary-value problems, one involving sublimation (condensation) at the inside surfaces of two parallel plates forming a flat channel with the length 2L, which is open on two sides, and sublimation (condensation) at the inside surfaces of a clearance between coaxial disks with the radius L. For these problems, Eq. (11) is reduced to the following form:

$$\frac{d}{d\xi_1}\left(\Phi_0\frac{d\Pi_1}{d\xi_1}\right) + \frac{\nu}{\xi_1}\Phi_0\frac{d\Pi_1}{d\xi_1} - \frac{\omega\ln\Pi_1}{1-\omega\ln\Pi_1} + \delta\Theta_w = 0. \tag{12}$$

We have $\nu = 0$ for the channel and $\nu = 1$ for the disks. The boundary conditions are

$$d\Pi_1/d\xi_1 = 0 \text{ for } \xi_1 = 0, \ \Pi_1 = p_0/p_* \text{ for } \xi = \sqrt{\omega_0}. \tag{13}$$

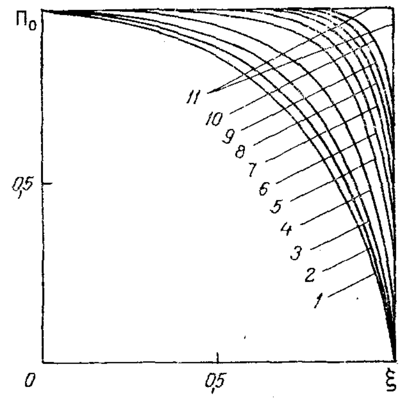

Fig. 3. Pressure distribution in the clearance between disks during water ice sublimation (ω = 0.04457) from the inside walls; ν = 1 and $K_{02} = K_{03} = 0$. 1) $\delta\Theta_w = 1$; 2) 10^{-1}; 3) 10^{-2}; 4) 10^{-3}; 5) 10^{-4}; 6) 10^{-5}; 7) 10^{-6}; 8) 10^{-7}; 9) 10^{-8}; 10) 10^{-9}; 11) 0.

If we use as the scale of pressure p_* its value for $\xi_1 = 0$, the boundary-value problem (12), (13) is reduced to the Cauchy problem for the boundary conditions

$$\Pi_1 = 1,\ d\Pi_1/d\xi_1 = 0 \text{ for } \xi_1 = 0. \tag{14}$$

In calculating sublimation devices, it is necessary to determine the limiting operating conditions corresponding to the maximum allowable temperature head $\delta\Theta_w$ producing at individual points temperatures which, when exceeded, lead to abnormalities in the technological process in connection with the unfreezing of the desublimate. Expression (7) is a monotonically increasing function, so that the above situation corresponds to an increase of the maximum pressure in the slot to its value at the triple point of the sublimable substance.

Problems (12), (14) pertain to the four-parameter set of functions $\Pi_1(\xi_1, \delta\Theta_w, \omega, K_{02}, K_{03})$. For Kn → 0, we have $K_{02}, K_{03} \to 0$, and the above set degenerates into the two-parameter set $\Pi_1(\xi_1, \delta\Theta_w, \omega)$. The parameter ω is considered to be a constant for each substance, since the temperature dependence Λ is not taken into account here. Generally speaking, there is another parameter, which is the exponent δ in the $\bar{\mu}(\Theta)$ relationship. In order to estimate the significance of this relationship for the processes in question, we considered two cases in calculations: the relative case corresponding to $\delta = 0(\mu = 1)$, and the one where $\delta = 1(\bar{\mu} = \Theta)$, which is in a certain sense the limiting case.

The results obtained by solving problem (12), (14) in the case of ω = 0.04457 (H_2O) for $\delta\Theta_w > 0$ (sublimation) and $\delta\Theta_w < 0$ (desublimation) are given in Figs. 1 and 2, respectively. If $|\delta\Theta_w| \ll 1$, phase transitions are localized in a relatively narrow zone adjacent to the open cutoff section of the slot system, i.e., the $\Pi_1(\xi_1)$ functions have in these cases the practically constant value $\Pi_1 \approx 1$ virtually throughout the range of values of the coordinate ξ_1, and they drop (increase) sharply only in the vicinity of the slot cutoff to a value equal to the external pressure if the latter is considerably different from 1, i.e., $\Pi_1(\sqrt{\omega_0}) \lessgtr 1$ ($\ll$ / $\gg$).

The localization of phase transitions in a relatively narrow peripheral zone whose extent is $\delta\xi_1 \ll \xi_1$ manifests itself in the fact that the presence of a central zone in the slot device is no longer perceived, and the line curvature is ξ = const for ν = 1. Therefore, the $\Pi_1(\xi_1)$ curves constitute virtually equidistant lines.

In Figs. 1 and 2, the points corresponding to the values $\delta = 1$, $\nu = 1$, and $K_{02} = K_{03} = 0$ lie very accurately on the solid curves corresponding to $\delta = 0$ for $\delta\Theta_w > 0$ and also for $\delta\Theta_w < 0$ and $\Pi_1 \lesssim 10$; consideration of the temperature dependence $\bar{\mu}(\Theta)$ within the $10 \lesssim \Pi_1 \lesssim 20$ range produces a shift to the left along ξ_1 by not more than 4%. All the curves in Figs. 1 and 2 are plotted for $\delta = 0$. Thus, consideration of the temperature dependence $\bar{\mu}(\Theta)$ hardly affects the pressure distribution in the slot space even for large relative pressure drop values, at any rate for substances characterized by small values of the parameter ω (of the H_2O type). The temperature dependence $\bar{\mu}(\Theta)$ can affect considerably the kinetics of phase transitions in the slot in the case of sublimation from one of the walls and heat supply to the other [4].

The diagram given in Fig. 1 can be used for practical calculations. For instance, if the channel extent (disk diameter) 2L, the pressure p_0 beyond the cutoff, and the effective heat transfer coefficients $\lambda^{(\pm)}/\delta^{(\pm)} = K^{(\pm)}$ are assigned, then, by equating the ratio p_0/p_* to $\Pi_1(\xi_1)$, we find $\omega_0 = \xi_1^2$ for different values of $\delta\Theta_w$ by using the appropriate curves and then calculate the clearance by means of the expression

$$h^3 = \frac{3}{2}\frac{\mu L^2 T_*}{p_* \omega_0}(K^{(-)} + K^{(+)}).$$

If p_* and T_* correspond to the triple point of the sublimate, unfreezing of the operating substance in the vicinity of $\xi_1 = 0$ will occur for smaller values of h.

Figure 3 shows a diagram for $\omega = 0.04457$, $\nu = 1$, and $K_{02} = K_{03} = 0$, which was obtained from that given in Fig. 1 by affine transformation: $\xi_1 \to \xi_{11} = \xi_1/\xi_m$, where ξ_m is the zero of the $\Pi_1(\xi_1)$ function. It is evident that, with an increase in $\delta\Theta_w$, the $\Pi_1(\xi_{11})$ profile tends, as its shape changes continuously, to a certain limiting configuration (extreme left) corresponding to $\delta\Theta_w \to \infty$. For $\omega = 0.04457$ and $\delta\Theta_w = 1$ the $\Pi_1(\xi_{11})$ profile is virtually identical to the limiting one. This self-similarity of the profile with respect to the $\delta\Theta_w$ parameter for sufficiently large values of the latter can be used for theoretical estimates if the limiting configuration and the $\xi_m(\delta\Theta_w)$ dependence have been plotted. Analysis of the processes is greatly simplified if $\omega \ll 1$, when the $\Pi_1(\xi_{11})$ profiles are self-similar with respect to ω as well. The limiting configuration is in this case the circular arc $\xi_{11}^2 + \Pi_1^2 = 1$, which is located in the first quadrant and corresponds to the solution of problem (12), (14) when $\delta\Theta_w \to \infty$, $\omega \to 0$, and $K_{02} = K_{03} = 0$ for $\nu = 0$ and $\nu = 1$. Curve 1 in Fig. 3 ($\delta\Theta_w = 1$) is in effect not different from a circular arc.

In conclusion, it should be noted that if there are sections at the open cutoff ends where the sublimate is supplied to the slot, we should, strictly speaking, take into account the pressure drop in the relatively narrows bands corresponding to sections of hydrodynamic stabilization of flow in slots [5]. However, for small values of the Kn number, this pressure drop is of the same order as Re_*, i.e., it can be neglected within the scope of our approximation.

NOTATION

$\bar{v}$, p, and T, velocity vector, pressure, and temperature of the sublimate, respectively; x, y, orthogonal coordinates in the median plane; z, distance from the median plane; Λ, latent heat of phase transition; R, gas constant; c_p and c_v, specific heat at constant pressure and constant volume, respectively; $\gamma_c = c_p/c_v$; μ and μ', the first and the second coefficient of dynamic viscosity, respectively; $\lambda^{(\pm)}$ and $\delta^{(\pm)}$, thermal conductivity coefficients and the wall thicknesses for the corresponding $z = \pm h$ values, respectively; $T^{(\pm)}$ and $J^{(\pm)}$, temperatures of the outside surfaces of these walls and the flux densities of the sublimate flow from them, respectively; 2h, magnitude of the clearance; L, characteristic linear scale in the median plane of the slot system; V, characteristic scale of the sublimate velocity; p_* and T_*, characteristic values of the sublimate pressure and temperature corresponding to a certain point on the saturation line; ℓ, mean free path of molecules; $\bar{v} = V(\bar{u} + \bar{e}_z \omega h/L)$; $\nabla = \bar{e}_x \partial/\partial\xi + \bar{e}_y \partial/\partial\eta$; $\nabla_1 = \bar{e}_x \partial/\partial\xi_1 + \bar{e}_y \partial/\partial\eta_1$; $x = L\xi$; $y = L\eta$; $z = h\zeta$; $\bar{e}_x$, $\bar{e}_y$, $\bar{e}_z$ unit vectors along the x, y, z axes; $\xi_1 = \sqrt{\omega_0}\xi$; $\eta_1 = \sqrt{\omega_0}\eta$; $\nabla = \omega_0 \nabla_1$; $\gamma = h/L$; $Kn = \ell/2h$, Knudsen number; $Re_* = pVh^2/\mu_* LRT_*$, reduced Reynolds number; $Pe = PrRe_*$, reduced Péclet number; $Pr = \mu_* c_p/\lambda_*$, Prandtl number; $M_* = Va_*$, characteristic value of the Mach number; $a_* = \sqrt{\gamma_c RT_*}$; $\Pi = ph^2/\mu_* LV$; $\Pi_* = p_* h^2/\mu_* LV$; $\Theta = T/T_*$; $\delta\Theta^{(\pm)} = \Theta^{(\pm)} - 1$; $\Theta_s(\Pi_1) = (1 - \omega \ln \Pi_1)^{-1}$; σ_0, percentage of molecules diffusely reflected from the wall; α, accommodation factor; $\omega = RT_*/\Lambda$; $J_0^{(\pm)} = (hRT_*/\mu_* V^2) \cdot J^{(\pm)}$; $J_\Lambda^{(\pm)} = (h\Lambda/\mu_* V^2) J^{(\pm)}$; $K^{(\pm)} = \lambda^{(\pm)}/\delta^{(\pm)}$; $\omega_0 = \frac{3}{2}\frac{\mu_* L^2 T_*}{h^3 p_*}(K^{(-)} + K^{(+)})$; $\delta\Theta_w = \frac{K^{(-)}\delta\Theta^{(-)} + K^{(+)}\delta\Theta^{(+)}}{K^{(-)} + K^{(+)}}$; $\Phi = \Pi_* \Phi_0$; $\Pi_1 = \Pi/\Pi_* = p/p_*$; $K_{02} = \frac{3{,}78\mu_* \Lambda}{hp_* \sqrt{RT_*}} \frac{2 - \sigma_0}{\sigma_0}$; $K_{03} = \frac{9\mu_*^2 \Lambda}{h^2 p_*^2}$; $K_2 = \omega K_{02}$; $K_3 = \omega K_{03}$; ∂/∂_N, derivative in the direction of the outward normal to the contour bounding the slot system; $\nu = 0$ or $\nu = 1$ for a flat channel or the clearance between disks, respectively; p_0, pressure beyond the cutoff section of the slot device; for $\nu = 1$, $\xi = r/L$, where r is the distance between the center of the disk and an arbitrary point at its surface; $\bar{\mu} - \mu/\mu_*$; $\bar{\mu}' = \mu'/\mu_*$; $\bar{\lambda} = \lambda/\lambda_*$.

LITERATURE CITED

1. P. A. Novikov, L. Ya. Lyubin, and V. I. Novikova, Inzh.-Fiz. Zh., 52, No. 1, 73-80 (1987).
2. L. D. Landau and E. M. Lifshits, Mechanics of Continuous Media [in Russian], Moscow (1954).

3. V. P. Shidlovskii, Introduction to Rarefied Gas Dynamics [in Russian], Moscow (1965).
4. P. A. Novikov, L. Ya. Lyubin, and V. I. Balakhonova, Inzh.-Fiz. Zh., 28, No. 1, 46-56 (1975).
5. T. S. Lungren, E. M. Sparrow, and D. Star, Teor. Osn. Inzh. Raschetov, Ser. D, 86, No. 3, 86-91 (1964).

HEAT-TREATMENT PROCESSES IN THE FORMATION OF COMPONENTS FROM POLYMER COMPOSITE MATERIALS

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UDC 678.5:539.42

A physicomathematical model of the heat treatment of components made from polymer composite materials on a rotating mandrel is considered.

With the expanding use of polymer composite materials (PCM) of constructional specification in various branches of the economy, the problem of retaining the initial properties of the components in the PCM is of great urgency. For example, in fiber-glass constructions obtained by the rolling method, only one third of the strength of the reinforcing fiber may be realized in practice [1]. In connection with this, it is necessary, first of all, to ensure optimal conditions for performing all stages of the technological process of producing PCM components: preparation of the initial components; soaking; formation of the components; solidification of the binder, etc.

The important factors influencing the final properties of the material obtained are the methods employed and the technology for thermal hardening of the polymer binder. The first largely determines the relative homogeneity of development of the temperature fields over the cross section of the component which forms, and the second the temperature–time characteristics of the process, which must ensure correspondence between the energy densities supplied and the physicochemical changes in composition occurring.

In realizing intensive conditions of PCM heat treatment in production, and in constructing and creating units in which effective thermoradiational–convective methods of energy supply are used, it is necessary to ensure "gradientless" (over the cross section of the component) heating in order to obtain high-quality components by methods of both wet and dry winding. In conection with the difficulty of experimental determination of temperature fields in PCM components, solidified on rotating mandrels, the mathematical modeling of such processes takes on practical importance.

The heat-conduction problem for systems of two cylindrical bodies is solved: components in the form of PCM tubes wound on a mandrel, rotating at specified frequency ω. The radiation sources are distributed on an arc s, as a result of which the component is subjected to pulsed heating. It is assumed that, if a point on the body falls under the radiation of the sources on arc s, then the pulse function $\varphi(\tau) = 1$, whereas if the point is in shadow then $\varphi(\tau) = 0$.

The mathematical model of this process comprises a system of equations of the form

$$\frac{\partial T}{\partial \tau} = a_2\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right), \quad R_3 \leqslant r \leqslant R_2, \tag{1}$$

$$\frac{\partial T}{\partial \tau} = a_1\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right), \quad R_2 \leqslant r \leqslant R_1, \tag{2}$$

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